Local Deviations from Uncovered Interest Parity: Kernel Smoothing Functions and the Role of Fundamentals

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Abstract

This paper uses recently developed kernel smoothing regression procedures and uniform confidence bounds to investigate the forward premium anomaly. These new statistical methods estimate the local time varying slope coefficient of the regression of spot returns on the lagged interest rate differential. The uniform confidence bands indicate the extent of the rejections of uncovered interest parity and find remarkable variation in both regimes when the anomaly occurs, and also the magnitude of the slope coefficient estimate. Of particular interest is the fact that the time varying slope parameter can be substantially explained by fundamentals such as monetary growth rates, and also the volatility of US money growth, which is associated with risk premium in many theoretical mdoels. Hence, the apparent deviations from uncovered interest parity have explanations consistent with monetary models and associated risk premium models.

Key words: Forward premium anomaly, Local Deviation from Uncovered Interest Parity, Time-varying parameters, Local-stationarity, Kernel smoothing, Local-linear regression, Uniform inference, Macroeconomic fundamentals.

JEL Classifications: C12; C14; C22; F31; F41; G15

1 Introduction

One of the long standing issues in international finance has been the apparent failure of the theory of Uncovered Interest Rate Parity (UIP). The classic method for testing UIPis to estimate the slope coefficient in a regression of spot returns on the lagged forward premium, or equivalently, the lagged interest rate differential. While the slope coefficient should be unity under UIP, most studies have found statistically significant rejections of the UIP hypothesis, with the slope coefficient estimate invariably being quite large and negative. This has become known as the *forward premium anomaly*. Hence most research has been directed at understanding the reasons for the apparent rejection of UIP and to try to account for it in terms of (i) time dependent risk premium, (ii) irrational agents and segmented markets, (iii) peso problems, or (iv) econometric issues with the testing of UIP. The dominant approach has been to explain the phenomenon by modeling a time dependent risk premium. Overall, this approach has not been particularly successful empirically.

The theory of UIP under rational expectations and a constant risk premium implies that

$$E_t(\Delta s_{t+1}) = (f_t - s_t) = (i_t - i_t^*) \tag{1}$$

is always an approximation which neglects the Jensen inequality terms, and possible time dependent risk premium. It has become standard to test the theory from the regression equation

$$\Delta s_{t+1} = \alpha + \beta (f_t - s_t) + u_{t+1}, \qquad (2)$$

where the theory of UIP implies $\alpha = 0$, $\beta = 1$ and u_{t+1} being serially uncorrelated¹.

¹Some studies such as Hansen and Hodrick (1980), Hakkio (1981) and Baillie, Lippens and Mcmahon (1983) tested the theory with overlapping data where the maturity time of the forward contract exceeds the sampling interval of the data. These studies still find rejection of UIP.

However, an increasing number of studies have come to recognize the fact that departures from UIP are more pronounced in some periods than others. The usual way of representing the potential variation in the slope coefficient is by rolling regressions, as in Baillie and Bollerslev (2000), Lothian and Wu (2011), etc. Other studies by Wolf (1987) have used Kalman filtering with the β following a random walk or stationary autoregression; while Bansal (1997) and Bansal and Dahlquist (2000) have allowed β to have two states depending on the sign of the interest rate differental; and Baillie and Kilic (2006) use a logistic smooth transiton regression to allow the β parameter to move slowly between the two states which correspond to either UIP holding², or alternatively a state with the forward premium anomaly being apparent. These parametric specifications for the time series behavior of the slope coefficient over time are necessarily heavily dependent on the parametric specification of the time series process for β_t .

While simple to apply in practice, the rolling regression technique is, however, highly arbitrary in the sense that the number of observations used in the window is very subjective. That is, there is no dependable criterion that one can use in choosing the right window size. The method also tends to produce quite wide confidence intervals from *OLS* regressions but does not allow any clear mehod for conducting statistical inference between different regressions.

²It should be noted that a more general representation of UIP is to begin with a standard discrete time, consumption based asset pricing model where the real returns of the representative investor are $E_t\left(\frac{S_{t+1}-F_t}{P_{t+1}}\right)\left(\frac{U'(C_{t+1})}{U'(C_t)}\right) = 0$, where S and F are the spot exchange rate and forward rate in levels, P is domestic price level and C is domestic consumption, and $U'(C_t)$ is the marginal utility of consumption in period t. Then,

$$E_t(\Delta s_{t+1}) = (f_t - s_t) - \left(\frac{1}{2}\right) Var_t(\Delta s_{t+1}) + Cov_t\left(\Delta s_{t+1}p_{t+1}\right) + \rho_t,$$
(3)

where ρ_t is the natural logarithm of the intertemporal marginal rate of substitution and is generally called the "risk premium". The above theory dates back at least to Hansen and Hodrick. (1983) One major novelty in this paper is to introduce the concept of Local Deviation from Uncovered Interest Parity (LDUIP), which is the specific amount that the parity condition is violated at each time point and is based on non-parametric and local smoothing techniques developed for the *local-linear regression* introduced by Stone (1977) and by Cleveland (1979). These techniques avoid the problems with rolling regressions and produce kernel smoothed regressions. They also allow *statistical inference* to be conducted on the parameters. The method assumes that the regression parameters are *smoothly varying* functions of time, and circumvents possible abrupt and sudden changes in the parameters. The method also enables the construction of *uniform confidence bands* (UCB) of the slope coefficient from its local-linear regression estimate. Hence the slope coefficient of the forward premium regression can be tested for any parametric specification of the unknown function. The generated LDUIP process and its associated UCB indicate the extent and significance of possible violations of UIP at any point of time.

A further interesting issue centers on the reasons for the changes in the relatively smooth pattern of the *LDUIP* and to what extent they can be predicted by macroconomic fundamentals and, or variables associated with time dependent risk premium. Some evidence is presented in section 4 of the paper that indicates a substantial role for lagged macroeconomic fundamentals and variables associated time risk premium, to have predictive power in explaining the movements of the time varying parameter in the forward premium regression.

The organization of the paper is the following: Section 2 introduces the model framework, and the forward premium regression with smoothly varying coefficients is explained. Section 3 then discusses the kernel smoothing regression and the construction of uniform confidenc bands (UCB) for inference. Section 4 presents the empirical results including the estimates of the time varying, estimated slope coefficients. The UCBs determine the precise time and extent of the violation of UIP for each currency over the sample period. This section also includes evidence from regression tests and VARs on the role of some fundamentals and financial variables that appear related to changes in the slope parameter of the forward premium regression. Section 5 concludes the paper and discusses related future research. The technical assumptions on the model and the details of the steps to construct the UCB are relegated to an appendix.

2 Model framework

It is first worth noting that apart from the strong empirical evidence, there are also economic reasons to allow the conventional regression in equation (2) to have time varying parameters. For example, the model

$$\Delta s_{t+1} = \alpha_t + \beta_t (f_t - s_t) + u_{t+1}, \tag{4}$$

can be justified from the approach of Chang (2013), where there is cross country speculation in stocks and bonds. Then with financial traders with negative exponential utility, the model implies a β_t that is the population equivalent of a regression of spot returns on lagged equity returns differentials. Extension of the model to include risk, or expectational error leads to a β_t that is the population equivalent of a regression of spot returns on a linear combination of variables associated with risk or expectational errors. Another motivation for time varying β_t could involve adaptation of the Taylor rule used in the exchange rate model of Engel and West (2005).

The approach taken on this paper is to be agnostic about the possible reasons for time variation in the slope coefficient and to essentially model it without any strong restrictions. Hence we estimate a model of the form

$$s_{\tau+1} - s_{\tau} = \beta_0 + \beta_1 (f_{\tau} - s_{\tau}) + \epsilon_{\tau+1}, \qquad \tau = 1, \cdots, T - 1.$$
(5)

where s_{τ} is the log of the monthly spot exchange rate at time τ , quoted as the foreign

price of domestic currency, while f_{τ} is the log of the corresponding 30 day forward rate, and finally $\epsilon_{\tau+1}$ is the stationary and serially uncorrelated disturbance term. In the above and throughout the paper, the index τ is reserved to indicate *discrete* time. Since the underlying unknown parameters β_0 and β_1 are specified as being deterministic and continuous functions of time, the traditional time index t is reserved for the *continuous* time processes in [0, 1]. Hence the following specification with *time-varying* parameters is then introduced to replace the traditional forward premium regression,

$$s_{\tau+1} - s_{\tau} = \beta_0 \left(\frac{\tau}{T}\right) + \beta_1 \left(\frac{\tau}{T}\right) (f_{\tau} - s_{\tau}) + \epsilon_{\tau+1}, \qquad \tau = 1, \cdots, T - 1.$$
(6)

where $\beta_0(\cdot)$ and $\beta_1(\cdot)$ are non-parametric functions of time which allows for potential time variation in the model parameters. Another advantage of this framework is that it enables *simultaneous inference* to be implemented for the unknown parameter functions.

Moreover, it is assumed that the regressors in (6) are *locally stationary* variables; see Dahlhaus (1997) and Kim, Zhou and Wu (2010). In a relatively short time span, they are approximately stationary. However, as the time horizon increases the variables show the characteristics of non-stationary processes, such as time-varying moments. One of the appealing features for this flexible class of non-stationarity is that its framework can encompass a wide range of popular linear or non-linear time series processes, such as various stationary processes or autoregressive processes with time varying parameters. Further details of the assumptions for (6) are given in Appendix 1.

One of the main features of the analysis in this paper is that, apart from modeling the time variation in the $\beta_1(\cdot)$, it also provides quite precise UCB that clearly indicate statistically significant departures from UIP. The precise reason for departures from UIPand their relationship to macroeconomic fundamentals and time depedent risk premium are investigated in section 4. Suffice to say, that these macro and financial variables are found to play an important role in the time varying nature of the slope coefficient estimate in the forward premium regression.

3 Methodology

The forward premium regression (6) with the smoothly time-varying coefficients, are estimated by using *kernel smoothing* techniques. This method allows one to perform estimation and inference of the true underlying process *without* imposing any arbitrary parametric assumptions on it. Thus, it minimizes the possibility of mis-specification. In addition, the method is computationally very tractable.

3.1 Local-linear regression

For simplicity, let $\Delta s_{\tau+1} := s_{\tau+1} - s_{\tau}$, $\mathbf{x}_{\tau}' := [1 \ f_{\tau} - s_{\tau}]$ and $\boldsymbol{\beta}(t)' := [\beta_0(t) \ \beta_1(t)]$, where $0 \le t \le 1$. Among various kernel smoothing techniques, the local-linear regression method of Stone (1977) and Cleveland (1979) stands out due to its simple form, ease of computation and analytical tractability. In contrast to other popular kernel smoothing methods, such as the Nadaraya-Watson estimator of Nadaraya (1964) and Watson (1964), the local-linear regression estimator does suppress the well-known *boundary problem* and achieve nearly optimal statistical efficiency; see Fan and Gijbels (1996). The local-linear regression estimates of the parameters in (6) are obtained by the following optimization:

$$(\hat{\boldsymbol{\beta}}(t), \hat{\boldsymbol{\beta}}'(t)) := \underset{(\eta_0, \eta_1)}{\operatorname{argmin}} \sum_{\tau=1}^{T-1} \left[\Delta s_{\tau+1} - \mathbf{x}_{\tau}' \eta_0 - \mathbf{x}_{\tau}' \eta_1 \left(t - (\tau/T) \right) \right]^2 K\left(\frac{t - (\tau/T)}{b} \right)$$
(7)

where $\hat{\boldsymbol{\beta}}(t)' = [\hat{\beta}_0(t) \ \hat{\beta}_1(t)]$ are the estimates of the model coefficients, $\beta_0(t)$ and $\beta_1(t)$, and $\hat{\boldsymbol{\beta}}'(t)' = [\hat{\beta}'_0(t) \ \hat{\beta}'_1(t)]$ are the estimates of their first-order derivatives, $\beta'_0(t)$ and $\beta'_1(t)$. Here $K(\cdot)$ is a kernel function and b is a bandwidth. In this study, we employ the Epanechnikov kernel $K(x) = 3 \max(1 - x^2, 0)/4$. The bandwidth is chosen by the generalized cross validation (*GCV*) procedure of Craven and Wahba (1979), and is described in detail in

section (i) of Appendix 2. The weak consistency of $\hat{\boldsymbol{\beta}}(t)$ given by (7) is proven in Theorem 1 of Kim, Zhou and Wu (2010):

$$\left\|\hat{\boldsymbol{\beta}}(t) - \boldsymbol{\beta}(t)\right\| = O\left(b^2 + \frac{1}{\sqrt{Tb}}\right)$$

where $b \to 0$, $Tb \to \infty$, and $\|\cdot\|$ is a norm defined by Appendix 1. Alternatively, one can instead consider the following *jackknife bias-corrected* estimator:

$$\tilde{\boldsymbol{\beta}}(t) := 2\hat{\boldsymbol{\beta}}_{b/\sqrt{2}}(t) - \hat{\boldsymbol{\beta}}(t) \tag{8}$$

where $\hat{\boldsymbol{\beta}}_{b/\sqrt{2}}(t)$ is $\hat{\boldsymbol{\beta}}(t)$ with $b/\sqrt{2}$ instead of the original bandwidth b.

A major motivation for using the kernel smoothed regression method in (7) comes from the literature that has investigated the time series properties of the forward premium. In particular, Baillie and Bollerslev (1994, 2000), Maynard and Phillips (2001) and Sakoulis, Zivot and Choi (2010) have found strong evidence that the forward premium, or equivalently the interest rate differential is a long memory, fractionally integrated process. There is also clear evidence that forward premium series are typically highly non-linear, with possible break points. Hence the general class of non-stationary, *locally stationary* process of Dahlhaus (1997), Kim, Zhou and Wu (2010) and Zhou and Wu (2010) seems an ideal assumption for dealing with this type of time series ³.

3.2 Uniform confidence band

One of the important advantages of the kernel smoothed regression in (7) is that the methodology allows the construction of *uniform confidence bands* (*UCB*) for the time varying parameters $\beta_0(\cdot)$ and $\beta_1(\cdot)$ in (6). Given the *UCB*, one can perform simultane-

³Other possible approaches for dealing with time varying parameters are to be found in the articles by Chen and Tsay (1993), Phillips (2001), Orbe, Ferreira and Rodrigues-Poo (2005, 2006) and Kim, Zhou and Wu (2010)

ous inference for the true underlying process, which facilitates the testing of the shape characteristics and subsequent tests for constancy, or linearity, etc.

In order to construct the asymptotic UCB of parameter $\beta_j(t)$, j = 0, 1, over $t \in [0, 1]$ with the confidence level $100(1 - \alpha)\%$, $\alpha \in (0, 1)$, one needs to find two functions $\ell_{j,n}(\cdot)$ and $u_{j,n}(\cdot)$ based on data, such that:

$$\lim_{n \to \infty} \mathbb{P}\{\ell_{j,n}(t) \le \beta_j(t) \le u_{j,n}(t) \text{ for } all \ t \in [0,1]\} = 1 - \alpha$$
(9)

The main purpose of constructing the UCB in (9) is to test whether the parameter $\beta_j(t)$ takes a certain parametric form. That is, using (9), we are able to test the null hypothesis $H_0: \beta_j(\cdot) = \beta_{j,\theta}(\cdot)$, where $\theta \in \Theta$ and Θ is a parameter space. For example, in order to test $H_0: \beta_j(t) = \theta_0 + \theta_1 t + \theta_2 t^2$, one can simply check whether $\ell_{j,n}(t) \leq \hat{\theta}_0 + \hat{\theta}_1 t + \hat{\theta}_2 t^2 \leq u_{j,n}(t)$ holds for all $t \in [0, 1]$. Here $\hat{\theta}_i$ is the least squares estimate of θ_i , i = 0, 1, 2. If the condition does hold for all $t \in [0, 1]$, then we fail to reject the null hypothesis at level α . In contrast, if the parametric fit of the function is not entirely covered by the constructed UCB, then the null is rejected.

The advantage of the UCB over the traditional point-wise bands is that the UCB can be used for simultaneous inference of an unknown function by allowing us to figure out the *overall* shape of the function. In addition, the UCB is a more *conservative* confidence band than the point-wise ones in that the UCB is *wider* than its point-wise copunterpart. Thus, any test results based on the UCB would be more *robust* than those under the point-wise ones. For these reasons, the UCB has recently gained more attention in econometrics and statistics literature. For example, Wu and Zhao (2007) construct the UCB for the trend of the global warming temperature series and show that the trend has been increasing. Zhao and Wu (2008) consider a discretized version of the stochastic diffusion model, and construct the UCBs of the conditional mean and volatility to test their parametric specifications. Zhou and Wu (2010) show how to construct the UCBs of time-varying regression coefficients and apply the method for the Hong Kong circulatory and respiratory data. Kim (2013a) applies the idea of UCB to the semi-parametric environmental Kuznets curve and test the hypothesis of an inverted U-shaped pattern for the income-pollution relationship. Furthermore, Kim (2013b) constructs the UCB of the long-run trend in unemployment rate, typically known as the NAIRU parameter of the Phillips Curve, and conduct simultaneous inference of the structural parameter.

Given these various interesting results, we shall construct the UCB of the slope coefficient (6), and carry out inference. To our best knowledge, this is the first time that one performs a UCB-based test on parametric specifications of the slope coefficient in the forward premium regression. Technical details of constructing the UCB are summarised in Appendix 2 of this paper.

4 Empirical results

The data used in this work are spot and one month forward exchange rate data involving the nine currencies of Australian Dollar (AUD), Canadian Dollar (CAD), Swiss Franc (CHF), Danish Krone (DKK), British Pound (GBP), Japanese Yen (JPY), Norwegian Krone (NOK), New Zealand Dollar (NZD), vis a vis U S Dollar (USD), ⁴ which is the numeraire currency in our study. This study uses end-of-month observations from December 1988 through October 2010.

The estimates of the time-varying slope coefficient and their corresponding 95% UCBs for the various currencies, are reported in Figures 1 and 2. The solid curves represent the local-linear regression estimates of the slope coefficients in (7), while the dashed bands are the 95% UCBs of the parameters. The optimal bandwidths for the local-linear regression

⁴The Euro that started in January 1999 is not used in this study to ensure that our sample is long enough for the non-parametric estimation and inference.

chosen by the GCV are also reported in the figures. The solid horizontal lines represent the hypothesis that $H_0: \beta_1(\cdot) = 1$. Figures 1 and 2 also present the OLS estimates of fixed β_1 in (5) for reference. In addition, Table 1 reports the proportion of $\hat{\beta}_1(t) > 1$ for each currency, where $\hat{\beta}_1(t)$ is the local-linear regression estimate of $\beta_1(t)$ in (6).

As seen from Figures 1 and 2, there is a substantial amount of time variation in the slope coefficients for all eight currencies, which are generally found to take both positive and negative values across time. Except for the CAD and GBP, the slope coefficients stay predominantly negative throughout the 1990s, which is consistent with studies such as Baillie and Bollerslev (2000) and Lothian and Wu (2011). However, there is some variation across currencies with the CAD having a positive slope coefficient during the second half of the 1990s; and also for the GBP which is briefly positive around 1995.

There is also considerable evidence that the the slope coefficients for all eight currencies, except for DKK, take *positive* values during from about 2007 onwards, which corresponds to the severe financial crisis. The slope coefficient for the AUD, CAD, CHF, JPY, NOK and NZD, becomes increasingly positive during the late 2000s, while the slope coefficient for the GBP is only barely positive during the period. This distinctive comovements among these slope coefficients is very pronounced during the crisis and highly suggestive of common factors determining their movements. Furthermore, after 2010, the slope coefficients for the DKK and GBP turn downward, while the coefficients for the other six currencies continue to climb upward.

From Fama (1984), it is well known that the slope coefficient of forward premium regression (5) can be expressed

$$\beta_{1} = \frac{Cov\left(p_{\tau}, q_{\tau}\right) + Var\left(q_{\tau}\right)}{Var\left(p_{\tau} + q_{\tau}\right)}$$

where $p_{\tau} = f_{\tau} - \mathbb{E}(s_{\tau+1}|\mathcal{I}_{\tau})$ is the risk premium and $q_{\tau} = \mathbb{E}(s_{\tau+1}|\mathcal{I}_{\tau}) - s_{\tau}$ is the expected rate of depreciation. Here \mathcal{I}_{τ} is the information set available up to time τ . Fama (1984) shows that $\beta_1 < 0$ requires both $Cov(p_{\tau}, q_{\tau}) < 0$ and $Var(p_{\tau}) > Var(q_{\tau})$. That is, if $\beta_1(t) < 0$ for some t, then both of these conditions are satisfied during that period. In contrast, if $\beta_1(t) > 0$, then at least one of these conditions are not satisfied. Figures 1 and 2 shows that throughout the financial crisis of late 2000s, we have either $Cov(p_{\tau}, q_{\tau}) > 0$ or $Var(p_{\tau}) < Var(q_{\tau})$ for all the currencies, except for the DKK. Typically, investors demand higher premium to keep holding depreciating currencies during financially unstable periods such as the late 2000s, which leads to the positive co-movement between the risk premium and expected depreciation during the period. This basic intuition appears to be well illustrated in Figures 1 and 2 by the positive β_1 's from the late 2000s.

4.1 Testing the UIP hypothesis

As previously explained, a great attraction of using the UCB of $\beta_1(\cdot)$ in this study is to carry out simultaneous inference of the unknown parameter function, and testing any hypothesis is simply facilitated by checking whether the specification is *contained* by the UCB. From Figures 1 and 2, it is easily seen that the null hypothesis is *rejected* at 5% for *all* eight currencies, because the solid horizontal line (*i.e.* $\beta_1 = 1$) cannot be entirely contained by the 95% UCBs. In fact, even more general hypothesis $H_0: \beta_1(\cdot) = c$, where c is some constant, and is also rejected at 5% for all the currencies, since no horizontal line can be placed entirely within the constructed 95% UCBs.

The Local Deviation from Uncovered Interest Parity (LDUIP) is given by

$$y_{\tau+1} = \Delta s_{\tau+1} - \widehat{\beta}_0 \left(\frac{\tau}{T}\right) - \widehat{\beta}_1 \left(\frac{\tau}{T}\right) \left(f_\tau - s_\tau\right)$$

and the percentage of the number of times that $\beta_1(\cdot) = 1$ is not covered or included by the *UCB*, is presented in Table 2. These results indicate that the percentage of occasions when $\beta_1(\cdot) > 1$ is 75% for Canada and around 40% for the UK, Norway and New Zealand. Table 3 shows corresponding percentages for when the direct *UIP* hypothesis $\beta_1(\cdot) = 1$ is violated. The hypothesis is rejected for 62% of the time periods for the Australian \$, and over 30% of the time for all currencies except Canada and Switzerland. Hence the empirical results confirm the forward premium anomaly for all the currencies for many of the time periods.

The key reason for the rejection of the null H_0 : $\beta_1(\cdot) = c$ is that the slope coefficient changes dramatically after the mid 2000s. For the AUD, CAD, CHF, JPY, NOK and NZD, the coefficients increase significantly during this period. On the other hand, β_1 for the DKK and GBP first increases during the early/mid 2000s, and then decreases, dramatically for the DKK and mildly for the GBP, during the late 2000s, which leads to the rejection of the null. Hence the approach in this study indicates the rejection of UIP for many time periods, and an additional attraction of the procedure is that there is clear co-movement among the slope coefficients for the different currencies. Large negative coefficients in the 1980s and 1990s are replaced, for many currencies, with a reversal of the forward premium anomaly after the financial crisis of 2008. This emphasizes the substantial variation in the slope coefficient and the inappropriateness of asserting that the anomaly exists for all time periods.

4.2 Determinants of the time varying beta: role of risk and fundamentals

Articles by Hansen and Hodrick (1983), Domowitz and Hakkio (1985), Hodrick (1987, 1989), Bekaert and Hodrick (1993), Baillie and Osterberg (1997), Mark and Wu (2000) and Verdelhan (2012) have all provided detailed models of time dependent risk premium, which have had variable degrees of empirical success. However, there has been no clear and definitive model of time-varying risk that has been found to be reliable across currencies and different time periods. The relatively smooth estimates of $\beta_1(\cdot)$ obtained by the kernel

weighted regresson show slow changes that turn out to be highly predictable from quite conventional models of time varying risk premium and also asset model funamentals. Given $\hat{\beta}_1(\cdot)$ from (6), the θ in (10) can be estimated by *OLS* and then a *Wald test* is used to test the hypothesis that H_0 : $\theta = 0$ versus H_1 : H_0 is incorrect. Hence a rejection of the hypothesis indicates that the violations of *UIP* can be at least partly explained by standard fundamentals and measures of risk. The following regression is then estimated,

$$\hat{\beta}_1 \left(\frac{\tau}{T}\right) - 1 = \mathbf{x}_{\tau-1}^{/} \theta + \xi_{\tau} \tag{10}$$

where θ is a parameter vector and $\hat{\beta}_1(\cdot)$ is the *non-parametric* estimate⁵ of $\beta_1(\cdot)$ in (6). Here ξ_{τ} is a mean zero error that is uncorrelated with $\mathbf{x}_{\tau-1}$. The covariates $\mathbf{x}_{\tau-1}$ in (10) are the following menu:

$$\mathbf{x}_{\tau-1}^{\prime} = \left[\Delta m_{\tau-1}, \Delta m_{\tau-1}^{*}, \Delta y_{\tau-1}, \Delta y_{\tau-1}^{*}, \left(i_{\tau-1}^{2} - i_{\tau-1}^{*2}\right), Var_{\tau-2}\Delta m_{\tau-1}\right]$$
(11)

where $\Delta m_{\tau-1}$ is the change in the log of the US money supply, $\Delta y_{\tau-1}$ is the change in the log of the US index of industrial production, (*) denotes foreign equivalents, while $(i_{\tau-1}^2 - i_{\tau-1}^{*2})$ is the differential between squared nominal 30 day T Bill interest rates for the relative volatility in the two bond markets; and $Var_{\tau-2}\Delta m_{\tau-1}$ represents the conditional variance of US money growth rates. This last variable was generated from a GARCH model, and is used following the findings in Hodrick (1989) and Baillie and Kilic (2006).

The Wald test statistics presented in Table 3 reveal substantial predictability of $\hat{\beta}_1(\cdot)$ from information in the lagged fundamentals and risk premium variables in (11). Similar analysis only based on information twelve months previously, as opposed to one month ago, is presented in Table 4. Again, the Wald test overwhelmingly rejects the null of no significance of the fundamentals for all cases for all eight currencies. The *R*-squared from the regressions reported in Tables 3 and 4 ranges from 15% to 35% over the eight currencies

⁵The estimate $\hat{\beta}_1(\cdot)$ will be discussed extensively in the following section.

being considered and indicate a substantial amount of predictability in the movement of the local $\hat{\beta}_1(\cdot)$ coefficient⁶.

The regression parameter estimates corresponding to the Wald tests in Tables 3 and 4 for equation (10) are reported in Tables 5 and 6, respectively. The impact of US money growth rates, and also the volatility of US money growth rates are seen to generally be statistically significant and positive for most of the currencies. The exception is for the Danish Krone, which moves in the opposite and non-intuitive direction. Overall, the effects also indicate substantial non-linearities with shocks on US money growth rates and its associated volatility leading to proprtional impacts of the interest rate differential on spot exchange rate returns.

4.3 Conclusion

This paper has used recently developed kernel smoothing regression procedures to derive uniform confidence bounds to investigate the forward premium anomaly, where spot currency returns are generally found to be negatively correlated with lagged interest rate diffrentials, or forward premium. The econometric techniques used in this paper have considerable advantages over simple rolling regression methods and they also provide relatively tight confidence intervals. The results indicate remarkable variation in the time periods where the anomaly occurs and where the deviatons from uncovered interest rate parity (UIP) are relatively small and fall within the bands of UIP. There is also some considerable similarity in *co-movements across currencies*. Hence, contrary to the established beliefs, the anomaly does not hold continuously and there are many time periods

⁶The inclusion of these variables in the standard rejection equation (10) would generally not lead to significant results due to the relatively very high volatility of spot returns tendings to dwarf the far smaller movements in the fundamentals. Hence our two-step analysis of the estimated beta slope coefficient has more economic interpretation.

where the hypothesis of *UIP* cannot be rejected. The departures from *UIP* throughout the sample were found to be partly predictable and to be based on many of the standard fundamentals associated with the monetary model, and also variables associated with time dependent risk premium.

As previously noted, the traditional models for time dependent risk premium and other explanations of the forward premium anomaly have not been very empirically successful. The results obtained in this study, suggest there is exploitable information in a function of the fundamentals and possible risk terms, which allow predictability of the extent, and even degree of persistence, of the anomaly. An interesting issue for future research is whether any existing models, or *combination of models*, might be consistent with this function of information, and how the success of the models varies over time. A full investigation of these issues is intriguing and is the subject of future research of the authors.

5 Appendix 1: Assumptions of Kernel Smoothing

First, we introduce notations. For any vector $\mathbf{v} = (v_1, v_2, \dots, v_p) \in \mathbb{R}^p$, we let $|\mathbf{v}| = (\sum_{i=1}^p v_i^2)^{1/2}$. For any random vector \mathbf{V} , we write $\mathbf{V} \in \mathcal{L}^q$ (q > 0) if $\|\mathbf{V}\|_q = [\mathbb{E}(|\mathbf{V}|^q)]^{1/q} < \infty$. In particular, $\|\mathbf{V}\| = \|\mathbf{V}\|_2$. We denote $\mathbf{L} : [0,1] \times \mathbb{R}^\infty \mapsto \mathbb{R}^p$ as a measurable function such that $\mathbf{L}(t, \mathcal{F}_i)$ is a properly defined $(p \times 1)$ random vector for all $t \in [0,1]$, where $\mathcal{F}_i = (\cdots, \eta_{i-1}, \eta_i)$ with independent and identically distributed (IID) random errors $\{\eta_j\}_{j\in\mathbb{Z}}$. Define the *physical dependence measure* (Wu, 2005) for $\mathbf{L}(t, \mathcal{F}_i)$ as the following:

$$\delta_q(\mathbf{L},k) = \sup_{t \in [0,1]} \|\mathbf{L}(t,\mathcal{F}_k) - \mathbf{L}(t,\mathcal{F}_k^*)\|_q$$
(12)

where $\mathcal{F}_i^* = (\cdots, \eta_0^*, \cdots, \eta_{i-1}, \eta_i)$ is a coupled process with η_0^* being an IID copy of η_0 . For discussion on this dependence measure, we refer to Wu (2005). For a class of stochastic processes $\{\mathbf{L}(t, \mathcal{F}_i)\}_{i \in \mathbb{Z}}$, we say that the process is \mathcal{L}^q stochastic Lipschitz-continuous over [0,1] if the following condition holds:

$$\sup_{0 \le t_1 < t_2 \le 1} \frac{\|\mathbf{L}(t_2, \mathcal{F}_0) - \mathbf{L}(t_1, \mathcal{F}_0)\|_q}{|t_2 - t_1|} < \infty$$
(13)

We denote a collection of such systems by Lip_q . The required assumptions are:

Assumption 1. Let covariates \mathbf{x}_{τ} of (7) be

$$\mathbf{x}_{\tau} = \mathbf{G}(\tau/T, \, \mathcal{U}_{\tau})$$

where $\mathbf{G} := (G_1, \cdots, G_p)^{\top}$ is a measurable function such that $\mathbf{G}(t, \mathcal{U}_{\tau})$ is well-defined for each $t \in [0, 1]$. Here $\mathcal{U}_{\tau} = (\cdots, u_{\tau-1}, u_{\tau})$ with IID errors $\{u_j\}_{j \in \mathbb{Z}}$. Moreover, $\mathbf{G}(t, \mathcal{U}_{\tau}) \in$ Lip_2 and $\sup_{0 \le t \le 1} \|\mathbf{G}(t, \mathcal{U}_{\tau})\|_4 < \infty$.

It should be noted that the Assumption 1 allows the regressors \mathbf{x}_{τ} to be *non-stationary* since their moments are allowed to be time-varying. Generally, the time variation in these characteristics is assumed to be *smooth*, rather than abrupt. Note also that \mathbf{x}_{τ} is *dependent* due to the cumulative IID random elements. Specifically, Assumption 1 ensures that model regressors \mathbf{x}_{τ} are *locally stationary* (Dahlhaus, 1997), which is a mild form of non-stationarity. That is, if one observes locally stationary variables in a relatively short time span, they are *approximately* stationary. However, in the long run, the variables behave as non-stationary ones. Since many economic variables are possibly locally stationary processes, we can make our model specification more general by introducing this assumption. For more on the local-stationarity, we refer to Priestley (1965), Dahlhaus (1997), Mallat, Papanicolaou & Zhang (1998), Ombao, Von Sachs & Guo (2005) and Kim, Zhou & Wu (2010).

Assumption 2. Let $M(t) := \mathbb{E} \left[\mathbf{G}(t, \mathcal{U}_0) \mathbf{G}(t, \mathcal{U}_0)^\top \right]$. Then, the smallest eigenvalue of

M(t) is bounded away from 0 on [0, 1].

This assumption prevents the asymptotic *multicollinearity* of regressors.

Assumption 3. The error ϵ_{τ} of (6) forms a stationary martingale-difference process such that

$$\epsilon_{\tau} = H(\mathcal{V}_{\tau})$$

where $H(\cdot)$ is a measurable function and $\mathcal{V}_{\tau} = (\cdots, v_{\tau-1}, v_{\tau})$ with IID errors $\{v_j\}_{j \in \mathbb{Z}}$. Here $\mathbb{E}(\epsilon_{\tau} | \mathbf{x}_{\tau}) = 0.$

Note that this assumption allows the error term to be *dependent* but *uncorrelated*. The dependence structure for ϵ_{τ} is flexible and general in that function $H(\cdot)$ is *not* specified. For example, a stationary *ARCH* process (Engle, 1982) would satisfy these requirements. Also, this assumption ensures that the error is uncorrelated with the regressors.

Assumption 4. Let $\mathbf{U}(t, \mathcal{I}_{\tau}) = \mathbf{G}(t, \mathcal{U}_{\tau}) H(\mathcal{V}_{\tau})$ where $\mathcal{I}_{\tau} = (\cdots, \zeta_{\tau-1}, \zeta_{\tau})$ and $\zeta_j = (u_j, v_j)^{/}$. Define

$$\Lambda(t) := \sum_{k \in \mathbb{Z}} cov \left(\mathbf{U}(t, \mathcal{I}_0), \, \mathbf{U}(t, \mathcal{I}_k) \right)$$

where the smallest eigenvalue of $\Lambda(t)$ is bounded away from 0 on [0, 1].

Assumption 5. Let $\sum_{\ell=0}^{\infty} [\delta_4(\mathbf{G}, \ell) + \delta_2(\mathbf{U}, \ell)] < \infty$.

This assumption also ensures *short-range dependence* among the variables in our model. The interpretation is that the cumulative effect of a single error on all future values is *bounded*. The measure of dependence used here is the *physical dependence measure* (Wu, 2005) based on causal processes. This measure is known to be particularly useful for characterizing dependence in *non-linear* time series models (Wu, 2005; Wu & Zhao, 2007; Kim, Zhou & Wu, 2010).

Assumption 6. The non-parametric functions $\beta_0(\cdot)$ and $\beta_1(\cdot)$ are twice continuously differentiable over the compact domain [0, 1].

This guarantees that the parameter functions $\beta_0(\cdot)$ and $\beta_1(\cdot)$ change smoothly over time. In particular, the second-order continuity of the parameters is required for the weak consistency of the local-linear estimates of their first-order derivatives.

Assumption 7. Let the kernel function $K(\cdot)$ be bounded, symmetric, with bounded support $[-A, A], K \in \mathcal{C}^1[-A, A], K(\pm A) = 0$ and $\sup_u |K'(u)| < \infty$.

This allows popular kernel functions such as the Epanechnikov kernel, which is used in the non-parametric estimation of $\beta_0(\cdot)$ and $\beta_1(\cdot)$.

6 Appendix 2: Construction of the UCB

Recall the bias-corrected estimator $\tilde{\boldsymbol{\beta}}(t)$, given by (8). Under Assumptions 1–7 and $Tb^7 \log(T) + \frac{(\log(T))^3}{bT^{2/5}} = o(1)$, Theorem 3 in Zhou and Wu (2010) shows:

$$\mathbb{P}\left\{\sqrt{\frac{Tb}{\lambda_K}}\sup_{t\in[0,1]} \left|\Sigma^{-1}(t)\left(\tilde{\boldsymbol{\beta}}(t)-\boldsymbol{\beta}(t)\right)\right| - d_T \le \frac{u}{\sqrt{2\log(b^{-1})}}\right\} \to e^{-2e^{-u}}$$
(14)

where $\lambda_K := \int_{\mathbb{R}} K^2(u) du$ and $\Sigma^2(t) := M^{-1}(t) \cdot \Lambda(t) \cdot M^{-1}(t)$ under Assumptions 2 and 4. Here the centering parameter d_T is defined by:

$$d_T := \sqrt{2\log(b^{-1})} + \frac{1/2\log(\log(b^{-1})) + 1/2\log\left(\int_{\mathbb{R}} (K'(u))^2 du/(4\pi\lambda_K)\right)}{\sqrt{2\log(b^{-1})}}$$

Note that the convergence rate to the asymptotic Gumbel distribution in (14) is $1/\sqrt{\log(T)}$, since $b \simeq T^{-\alpha}$, $\alpha \in (0, 1)$. Given this slow rate of convergence, we employ the following *invariance principle* (Zhou and Wu, 2010):

$$\sqrt{\frac{Tb}{\log(T)}} \sup_{t \in [0,1]} \left| \tilde{\boldsymbol{\beta}}(t) - \boldsymbol{\beta}(t) - \boldsymbol{\Sigma}(t) \sum_{\tau=1}^{T} w_T^*(t,\tau) \mathbf{Z}_{\tau} \right| = o_{\mathbb{P}}(1)$$
(15)

where $\mathbf{Z}_{\tau} \sim NID(\mathbf{0}, \mathbf{I}d_2)$ and $w_T^*(t, \tau) = \frac{1}{Tb}K^*\left(\frac{t-\tau/T}{b}\right)$ with $K^*(u) = 2\sqrt{2}K(\sqrt{2}u) - K(u)$ from the bias-correction. Here $\Sigma(t)$ is introduced by (14). Then, by (15), it is then possible to construct the *UCB* of the slope coefficient $\beta_1(t), 0 \leq t \leq 1$, in (6). The procedure requires the following steps:

(i) (Bandwidth selection) Consider $\hat{\beta}(t)$ in (8) under some *b* and the Epanechnikov kernel. Then, the fitted values for (6) would be $\widehat{\Delta s_{\tau+1}}(b) = \tilde{\beta}_0(\tau/T) + \tilde{\beta}_1(\tau/T)(f_\tau - s_\tau), \quad \tau = 1, \dots, T-1$. Note here that $\widehat{\Delta s_{\tau+1}}(b)$ depends on *b* due to $\tilde{\beta}_0(\tau/T)$ and $\tilde{\beta}_1(\tau/T)$. To pick up the *optimal* bandwidth, we consider:

$$S(b) = H(b)Y,$$

where $S(b) := \left(\widehat{\Delta s_1}(b), \widehat{\Delta s_2}(b), \cdots, \widehat{\Delta s_T}(b)\right)^{/}$, $Y := (\Delta s_1, \cdots, \Delta s_T)^{/}$ and H(b) is a $(T \times T)$ smoothing matrix that depends on b. The GCV criterion (Craven and Wahba, 1979) chooses the optimal bandwidth b^{opt} that minimizes the following criterion:

$$b^{opt} := \underset{b}{\operatorname{argmin}} \frac{T^{-1} \sum_{\tau=1}^{T} \left(\Delta s_{\tau} - \widehat{\Delta s_{\tau}}(b) \right)^{2}}{\left\{ 1 - \operatorname{trace}[H(b)]/T \right\}^{2}}$$

where the numerator represents the goodness-of-fit and the denominator can be viewed as the model's degrees of freedom (Kim, Zhou & Wu, 2010). We obtain (8) using b^{opt} .

(ii) Compute $\sup_{0 \le t \le 1} \left| \sum_{\tau=1}^{T} w_T^*(t,\tau) Z_\tau \right|$, where $\{Z_\tau\}$ are generated as NID(0,1) random variables and $w_T^*(t,\tau) = \frac{1}{Tb} K^*\left(\frac{t-\tau/T}{b}\right)$ with higher-order kernel $K^*(u) = 2\sqrt{2}K(\sqrt{2}u) - \frac{1}{Tb}K^*\left(\frac{t-\tau/T}{b}\right)$

K(u).

(iii) Repeat (ii), say 1,000 times, and then obtain the 95th quantile of the sampling distribution of $\sup_{0 \le t \le 1} \left| \sum_{\tau=1}^{T} w_T^*(t,\tau) Z_{\tau} \right|$, and denote it as $\hat{q}_{0.95}$.

(iv) Estimate $\Sigma^2(t)$ by:

$$\hat{\Sigma}^2(t) := \hat{M}^{-1}(t) \cdot \hat{\Lambda}(t) \cdot \hat{M}^{-1}(t)$$

where $\hat{M}(t)$ and $\hat{\Lambda}(t)$ are the estimates of M(t) and $\Lambda(t)$ in Assumptions 2 and 4. The estimates are provided by Section 4.3 of Zhou and Wu (2010). Denote the (i, j)th element of $\hat{\Sigma}^2(t)$ by $\hat{\sigma}_{i,j}^2(t)$.

(v) The 95% *UCB* of
$$\beta_1(t)$$
 is $\left[\tilde{\beta}_1(t) \pm \hat{q}_{0.95}\hat{\sigma}_{2,2}(t)\right]$.

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Figure 1: The *solid* curve is the local-linear regression estimate of $\beta_1(\cdot)$ in (6). The *dashed* band is the 95% *UCB* of $\beta_1(\cdot)$ and the solid horizontal line is H_0 : $\beta_1(\cdot) = 1$. The *dot-dashed* horizontal line is the *OLS* estimate of fixed β_1 in (5). The *GCV*-chosen bandwidths are (a) 0.35 (b) 0.36 (c) 0.28 (d) 0.30.



Figure 2: The *solid* curve is the local-linear regression estimate of $\beta_1(\cdot)$ in (6). The *dashed* band is the 95% *UCB* of $\beta_1(\cdot)$ and the solid horizontal line is H_0 : $\beta_1(\cdot) = 1$. The *dot-dashed* horizontal line is the *OLS* estimate of fixed β_1 in (5). The *GCV*-chosen bandwidths are (a) 0.33 (b) 0.27 (c) 0.27 (d) 0.30.

Table 1: Proportion of $\hat{\beta}_1(t) > 1$				
Currency	Proportion			
Australian Dollar (AUD)	0.2252			
Canadian Dollar (CAD)	0.7557			
Swiss Franc (CHF)	0.1832			
Danish Krone (DKK)	0.0763			
British Pound (GBP)	0.3931			
Japanese Yen (JPY)	0.1489			
Norweigian Krone (NOK)	0.4046			
New Zealand Dollar (NZD)	0.4504			

Currency Proportion Australian Dollar (AUD) 0.6260Canadian Dollar (CAD) 0.1565Swiss Franc (CHF) 0.1489Danish Krone (DKK) 0.3015British Pound (GBP) 0.2023Japanese Yen (JPY) 0.4122Norweigian Krone (NOK) 0.3969New Zealand Dollar (NZD) 0.4084

Table 2: Proportion of $\beta_1 = 1$ violating 95% UCB

Currency	Wald statistic	p-value
Australian Dollar (AUD)	53.25	< 0.001
Canadian Dollar (CAD)	85.72	< 0.001
Swiss Franc (CHF)	30.19	< 0.001
Danish Krone (DKK)	99.87	< 0.001
British Pound (GBP)	140.23	< 0.001
Japanese Yen (JPY)	111.06	< 0.001
Norweigian Krone (NOK)	51.21	< 0.001
New Zealand Dollar (NZD)	36.62	< 0.001

Table 3: Wald statistic and p-value for (10) with lag=1

Table 4: Wald statistic and p-value for (10) with lag=12

Currency	Wald statistic	p-value
Australian Dollar (AUD)	49.11	< 0.001
Canadian Dollar (CAD)	81.58	< 0.001
Swiss Franc (CHF)	28.28	< 0.001
Danish Krone (DKK)	85.32	< 0.001
British Pound (GBP)	51.13	< 0.001
Japanese Yen (JPY)	39.19	< 0.001
Norweigian Krone (NOK)	49.44	< 0.001
New Zealand Dollar (NZD)	45.65	< 0.001

Currency	US money	Foreign money	US production	Foreign production	T-Bill differential	US money volatility
AUD	-32.88 (0.4802)	$\underset{(0.0037)}{-46.28}$	-18.24 (0.3002)	$\underset{(0.0045)}{-165.8}$	$\underset{(3.68\times10^{-6})}{\textbf{0.0279}}$	$\underset{(0.6525)}{6.6\times10^{-4}}$
CAD	$\underset{(0.0367)}{\textbf{126.7}}$	$\underset{(0.0120)}{\textbf{84.26}}$	$\underset{(0.4102)}{18.97}$	$\underset{(0.3153)}{79.67}$	$\substack{\textbf{0.0363}\\(0.0248)}$	$\underset{(2.92\times10^{-9})}{\textbf{0.0114}}$
CHF	$\underset{(0.8783)}{-10.52}$	$\underset{(0.3304)}{25.84}$	-3.89 (0.8770)	-3.87 (0.8027)	$-{0.0887 \atop (0.0003)}$	0.0076 (0.0007)
DKK	$\underset{(0.0395)}{-60.75}$	$\underset{(0.0043)}{-17.99}$	$\underset{(0.0006)}{-54.97}$	$\underset{(0.0366)}{\textbf{4.29}}$	$-6.97 \times 10^{-3}_{(0.1333)}$	$\substack{-6.63\times10^{-3}\\(4.22\times10^{-13})}$
GBP	$\mathop{74.46}_{(0.0077)}$	-18.52 (0.2822)	$\underset{(0.7352)}{3.59}$	$\underset{(0.7378)}{1.045}$	$\underset{(<2.00\times10^{-16})}{\textbf{0.0461}}$	-2.69×10^{-3}
JPY	-69.45 $_{(0.1183)}$	-9.98 (0.2185)	$\underset{(0.4257)}{-13.46}$	$\underset{(0.6056)}{2.258}$	$\begin{array}{c} -0.1539 \\ \scriptscriptstyle (<\!\!2.00\times10^{-16}) \end{array}$	0.0053 (0.0003)
NOK	$\underset{(0.0018)}{\textbf{237.25}}$	$\underset{(0.9166)}{3.5596}$	$\underset{(0.2998)}{35.35}$	-9.999 (0.2228)	$\underset{(5.57\times10^{-6})}{\textbf{0.0658}}$	$\substack{\textbf{0.0113}\\(1.93\times 10^{-6})}$
NZD	$\underset{(0.0343)}{\textbf{182.03}}$	$\underset{(0.8389)}{8.9843}$	$\underset{(0.6910)}{13.08}$	-26.64 (0.2997)	$\begin{array}{c} 0.0247 \\ (0.1228) \end{array}$	$\underset{(1.56\times10^{-5})}{0.0122}$

Table 5: Estimates of the coefficients in (10) when lag=1; The numbers inside the brackets are the corresponding p-values. The coefficients significant at 5% are in bold.

Currency	US money	Foreign money	US production	Foreign production	T-Bill differential	US money volatility
AUD	-53.79 $_{(0.2608)}$	$\underset{(0.0095)}{-41.62}$	-26.54 $_{(0.1359)}$	-186.8 (0.0020)	$\underset{(2.42\times10^{-5})}{\textbf{0.0256}}$	$1.91 \times 10^{-3} \\ _{(0.2094)}$
CAD	$\underset{(0.0316)}{\textbf{136.76}}$	${f 95.57}_{(0.0064)}$	$\underset{(0.7970)}{6.0913}$	-10.21 (0.9006)	$0.0384 \\ (0.0206)$	$\underset{(7.08\times10^{-9})}{\textbf{0.0117}}$
CHF	$\underset{(0.6383)}{33.87}$	$\underset{(0.3609)}{24.87}$	-28.57 $_{(0.2697)}$	-6.72 (0.6761)	$-{0.0662 \atop \scriptstyle (0.0083)}$	$\substack{\textbf{0.0075}\\(0.0014)}$
DKK	$\underset{(0.1591)}{-43.28}$	$-{16.74 \atop \scriptscriptstyle (0.0100)}$	$\underset{(0.0009)}{-55.99}$	$\mathop{\bf 4.62}_{(0.0326)}$	$-\underset{(0.0201)}{\textbf{0.0201)}}\textbf{-0.0111}$	$-{\color{red}{6.78}}_{\scriptstyle{(1.97\times10^{-12})}}{\color{red}{\times10^{-3}}}$
GBP	$\underset{(0.0001)}{\textbf{107.9}}$	$\underset{(0.0124)}{-49.95}$	$\underset{(0.6512)}{4.72}$	$\underset{(0.5895)}{1.682}$	$\substack{\textbf{0.0173}\\(0.0002)}$	-3.10×10^{-3}
JPY	$\underset{(0.0352)}{-106.9}$	$\underset{(0.3136)}{-9.116}$	$\underset{(0.1761)}{-25.62}$	$\underset{(0.5973)}{2.619}$	-0.0956 (3.90×10 ⁻⁸)	${\color{red}{4.17\times10^{-3}}\atop_{\scriptstyle{(0.0123)}}}$
NOK	$\underset{(0.0006)}{\textbf{274.63}}$	$\underset{(0.4927)}{23.90}$	$\underset{(0.6564)}{-15.55}$	-1.88 (0.8240)	$\substack{\textbf{0.0507}\\(0.0006)}$	${\displaystyle {f 0.0112}\atop {}_{(7.8 imes 10^{-6})}}$
NZD	$\underset{(0.0351)}{\textbf{181.94}}$	$\underset{(0.8091)}{10.84}$	-15.99 $_{(0.6224)}$	-26.71 (0.3016)	-0.0158 $_{(0.3164)}$	${\substack{0.0116}\atop{(5.6 imes 10^{-5})}}$

Table 6: Estimates of the coefficients in (10) when lag=12; The numbers inside the brackets are the corresponding p-values. The coefficients significant at 5% are in bold.